Granular Reasoning Using Zooming In & Out

Part 2. Aristotle's Categorical Syllogism

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Abstract

The concept of granular computing is applied to Aristotle's categorical syllogism. Such kind of reasoning is called granular reasoning in this paper. For the purpose, two operations called zooming in & out is introduced to reconstruct granules of possible worlds.

Key words: Granular reasoning, zooming in & out, filtration, Aristotle's syllogism.

1 Introduction

Recently, Lin[2], Skowron[9], and others have developed granular computing based on rough set theory (Pawlak [6,7]) and many researchers expect that it provides a new paradigm of computing. In this paper, by *granular reasoning*, we mean some mechanism for reasoning using granular computing. We described in [3] a possible step for granular reasoning using filtration in modal

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logic[1]. Then, in [5], we applied the idea to propositional reasoning using two operations called zooming in & out proposed in [4]. This paper aims to provide the next step for formulating granularity of an aspect of Aristotle's syllogism.

But why now Aristotle? In our opinion, Aristotle's categorical syllogism may capture the essence of human ordinary reasoning, not of mathematical reasoning. We cannot forget the fact that the great work of F regeprovides a precise expression of mathematical inference, but when it was applied to h u-man ordinary reasoning, we had to face some very difficult problems including the *frame problem.* Frege analyzed a universal sentence *'(All)* s *is* p*'* using his inv en tion*universal quantifier* as $\forall x(\mathsf{s}(x) \rightarrow \mathsf{p}(x))$. His analysis is undoubtedly correct and was the motive power of a great deal of brilliant results in mathematical logic since the 20th century. At the same time, nevertheless, his analysis was too much detailed for human beings, in general, to carry out their ordinary reasoning in his fashion. It is well-known that Frege analyzed sentences in a structure of 'individual—predicate,' while Aristotle's structure was 'subject—predicate.' Clearly, however, we do not hav e to analyze every concept at the level of individuals every time for ordinary reasoning. F rege requires us complete analysis, which may cause intractability, while the basis of Aristotle's logic seems to be granules.

This paper aims to give a step for describing Aristotle's syllogism in a wa y of granular computing. Let $\mathrm{M} = <U, I>$ be a structure for predicate logic, where U is a domain and I is an interpretation. In rough set theory, sentences in predicate logic is often represented in a Kripke-st yle model $\mathfrak{M}$, where a domain and predicate symbols are regarded as a set of worlds and atomic sentences, respectively, and thereby we hav e, for example, the follwing correspondence

$$\mathrm{M} \models \mathsf{mortal}(\mathsf{socrates}) \ \text{ iff } \ \mathfrak{M}, \mathsf{socrates} \models \mathsf{mortal}. \tag{1}$$

Here make a quotient model $\mathfrak{M}/\sim_{\mathsf{human}}$ using an equivalence relation $\sim_{\mathsf{human}}$ which means two worlds are equivalent just in case they both hav e the property 'human' in common. Then one equivalent class must be the set of humans denoted b y $I(\mathsf{human})$, where I is an interpretation in predicate logic. F or example, $I(\mathsf{human})=[\mathsf{socrates}]_{\sim_{\mathsf{human}}}$ holds by taking $\mathsf{socrates}$ as a representative element. Thus, in the quotient model, for instance, we may write

$$\mathfrak{M}/\sim_{\mathsf{human}}, I(\mathsf{human}) \models \mathsf{mortal}. \tag{2}$$

Can we see this expression (2) as corresponding to Aristotle's analyses *'(All)* human *is* mortal*'*? This may suggest a possible way of formulating higher-order predicate expressions lik e $\mathsf{mortal}(\mathsf{human})$ under, for instance, the following correspondence: for some structure M',

$$\mathrm{M}' \models \mathsf{mortal}(\mathsf{human}) \ \overset{?}{\text{iff}} \ \mathfrak{M}/\sim_{\mathsf{human}}, I(\mathsf{human}) \models \mathsf{mortal},$$

which is parallel to formula (1). This is the starting point for our research. Then, in such an approach, we need reasoning process using reconstruction of models. We call such operations 'zooming in & out' in [4], which is defined based on some idea of granularity introduced to give a logical foundation of local and global worlds in semantic fields[8]. Such operators provides us a way of changing our viewpoint moving from global to local and vice verse. In this paper, using such operations of zooming in & out, we try to describe Aristotle's categorical syllogism as a small step for formulating granular reasoning.

2 Granules of Possible Worlds

Given a countably infinite set of atomic sentences $\mathfrak{P}$, a language $\mathfrak{L}_{\mathrm{BL}}(\mathfrak{P})$ for propositional logic of belief is formed as the least set of sentences from $\mathfrak{P}$ with the well-known set of connectives with a modal operator B(belief) by the usual formation rules. A sentence is called *non-modal* if it does not contain any occurrence of B. A Kripke model is a tuple $\mathfrak{M}$=$<U, R, V>$, where W is a non-empty set of (possible) worlds, R is a binary relation on W, and V is a valuation for every atomic sentence p at every world x. Define $\mathfrak{M}, x \models \mathsf{p}$ iff $V(\mathsf{p}, x)$=1. The relationship $\models$ is extended for every compound sentence in the usual way. The *truth set* of p in $\mathfrak{M}$ is define as $\|p\| =\{x \in U \mid \mathfrak{M}, x \models p\}$. A sentence p is said to be *valid* in $\mathfrak{M}$, written $\mathfrak{M} \models p$, just in case $\|p\| = U$.

Given $\mathfrak{P}$, let U be a non-empty subset[5] of 2 . We call any subset in U an *elementary world* in U. Then, for an atomic sentence p in $\mathfrak{P}$ and an elementary world x in U, a *valuation* V is naturally defined by $V(\mathsf{p}, x)$=1 iff $\mathsf{p} \in x$. When a binary relation R is given on U, we have a Kripke model $\mathfrak{M}$=$<U, R, V>$. When we are concerned only with finite sentences, the set U is, in general, large for us. Hence we need some way of granularizing U. Our proposal in a series of papers[3,4,5] is to make a quotient set whose elements we regard as granules of possible worlds. Suppose we are concerned with a set Γ of non-modal sentences. Let $\mathfrak{P}_\Gamma = \mathfrak{P} \cap \mathrm{sub}(\Gamma)$, where $\mathrm{sub}(\Gamma)$ is the union of the sets of subsentences of each sentence in Γ. Then, we can define an agreement relation $\sim_\Gamma$ by $x \sim_\Gamma y$ iff $\forall \mathsf{p} \in \mathfrak{P}_\Gamma [V(\mathsf{p}, x) = V(\mathsf{p}, y)]$. The relation becomes an equivalence relation and induces the quotient set $U_\Gamma \stackrel{\mathrm{df}}{=} U/\sim_\Gamma$. We regard its (non-empty) elements as the granules of possible worlds under Γ. A new valuation is given by $V_\Gamma(\mathsf{p}, X)$=1 iff $\mathsf{p} \in \cap X$, for p in $\mathfrak{P}_\Gamma$ and X in U_Γ. According to [1], when a relation R is given on U, assume we have an accessibility relation R' on U_Γ satisfying (a) if xRy then $[x]_{\sim_\Gamma} R' [y]_{\sim_\Gamma}$, (b) if $[x]_{\sim_\Gamma} R' [y]_{\sim_\Gamma}$ then $\mathfrak{M}, x \models \mathsf{B}p \Rightarrow \mathfrak{M}, y \models p$, for every sentence $\mathsf{B}p$ in Γ, and (c) if $[x]_{\sim_\Gamma} R' [y]_{\sim_\Gamma}$ then $\mathfrak{M}, y \models p \Rightarrow \mathfrak{M}, x \models \neg\mathsf{B}\neg p$, for every sentence $\neg\mathsf{B}\neg p$ in Γ, then the model $\mathfrak{M}_\Gamma^{R'}$=$<U_\Gamma, R', V_\Gamma>$ is called a *filtration* through $\mathrm{sub}(\Gamma)$.

[5] More generally, we may take a *multisubset* of 2 as a set of possible worlds.

3 Zooming In & Out

3.1 Zooming In & Out on Sets of Worlds.

Let Γ be a set of non-modal sentences we are concerned with at a given time. It is called a *focus* at the time. When we move our viewpoint from one focus to another along time, we must reconstruct the set of granularized possible worlds. Let Γ be the current focus and Δ be the next focus we will move to. First we consider the simpler two nested cases.

(a) When $\mathfrak{P}_\Gamma \supseteq \mathfrak{P}_\Delta$, we need granularization, which is represented by a mapping $\mathfrak{I}^\Gamma_\Delta$: $U_\Gamma \to U_\Delta$, called a *zooming in from* Γ *to* Δ, where, for any X in U_Γ, $\mathfrak{I}^\Gamma_\Delta(X) \stackrel{\mathrm{df}}{=} \{x \in U \mid x \cap \mathfrak{P}_\Delta = (\cap X) \cap \mathfrak{P}_\Delta\}$.

(b) When $\mathfrak{P}_\Gamma \subseteq \mathfrak{P}_\Delta$, we need an inverse operation of granularization $\mathfrak{O}^\Gamma_\Delta$: $U_\Gamma \to 2^{U_\Delta}$, called a *zooming out from* Γ *to* Δ, where, for any X in U_Γ, $\mathfrak{O}^\Gamma_\Delta(X) \stackrel{\mathrm{df}}{=} \{Y \in U_\Delta \mid (\cap Y) \cap \mathfrak{P}_\Gamma = \cap X\}$.

(c) For non-nested two sets Γ, Δ, the movement from Γ to Δ can be represented using combination of 'zooming out & in' as $\mathfrak{I}^{\Gamma \cup \Delta}_\Delta \circ \mathfrak{O}^\Gamma_{\Gamma \cup \Delta} : U_\Gamma \to 2^{U_\Delta}$.

3.2 Extending zooming in & out on models.

We extend the two operations so that they can be applied to models. Again let Γ and Δ be the current and the next focus, respectively. Given a model $\mathfrak{M}_\Gamma = <U_\Gamma, R_\Gamma, V_\Gamma>$, we define, for X in U_Γ, $B_\Gamma(X) \stackrel{\mathrm{df}}{=} \{X' \in U_\Gamma \mid X R_\Gamma X'\}$. We abbreviate it to B_Γ, when $B_\Gamma(X) = B_\Gamma(X')$ for any X, X' in U.

(a) When $\Gamma \supseteq \Delta$, a *zooming in of* $\mathfrak{M}_\Gamma$ *through* Δ is a tuple $\mathfrak{I}^\Gamma_\Delta(\mathfrak{M}_\Gamma) \stackrel{\mathrm{df}}{=} <U_\Delta, R_\Delta, V_\Delta>$, where $Y_i R_\Delta Y_j$ iff $Y_j \in \mathfrak{I}^\Gamma_\Delta(\cup\{B_\Gamma(X) \mid X \in U_\Gamma \text{ and } \mathfrak{I}^\Gamma_\Delta(X) = Y_i\})$.

(b) When $\Gamma \subseteq \Delta$, a *zooming out of* $\mathfrak{M}_\Gamma$ *through* Δ is a tuple $\mathfrak{O}^\Gamma_\Delta(\mathfrak{M}_\Gamma) \stackrel{\mathrm{df}}{=} <U_\Delta, R_\Delta, V_\Delta>$, where $Y_i R_\Delta Y_j$ iff $Y_j \in \mathfrak{O}^\Gamma_\Delta(B_\Gamma((\mathfrak{O}^\Gamma_\Delta)^{-1}(Y_i)))$.

(c) Non-nested cases are described using a *merging* of two models $\mathfrak{M}_\Gamma$ and $\mathfrak{M}_\Delta$. When $\mathfrak{P}_\Gamma = \mathfrak{P}_\Delta$, their merging $\mathfrak{M}_\Gamma \circ \mathfrak{M}_\Delta$ is $<U_\Gamma, R, V_\Gamma>$, where $X_i R X_j$ iff $X_j \in B_\Gamma(X_i) \cap B_\Delta(X_i)$. The merging $\circ$ is extended for the cases that $\mathfrak{P}_\Gamma \neq \mathfrak{P}_\Delta$: If $\mathfrak{P}_\Gamma \supset \mathfrak{P}_\Delta$, then $\mathfrak{M}_\Gamma \circ \mathfrak{M}_\Delta \stackrel{\mathrm{df}}{=} \mathfrak{M}_\Gamma \circ \mathfrak{O}^\Delta_\Gamma(\mathfrak{M}_\Delta)$, else if $\mathfrak{P}_\Gamma \subset \mathfrak{P}_\Delta$, then $\mathfrak{M}_\Gamma \circ \mathfrak{M}_\Delta \stackrel{\mathrm{df}}{=} \mathfrak{O}^\Gamma_\Delta(\mathfrak{M}_\Gamma) \circ \mathfrak{M}_\Delta$, else $\mathfrak{M}_\Gamma \circ \mathfrak{M}_\Delta \stackrel{\mathrm{df}}{=} \mathfrak{O}^\Gamma_{\Gamma \cup \Delta}(\mathfrak{M}_\Gamma) \circ \mathfrak{O}^\Delta_{\Gamma \cup \Delta}(\mathfrak{M}_\Delta)$. The last case of merging is used for non-nested cases.

4 Aristotle's Syllogism and Granularity

4.1 Preliminaries

We confine ourselves to a monadic predicate logic. Let $\mathfrak{P}$ be a non-empty set of predicate symbols of arity 1 with $\top$ and $\bot$. Let C be a non-empty set of constants. For a structure M$=<U, I>$ for a monadic predicate logic, where U is a domain and I is an interpretation, define a mapping ψ: $U \to 2$ by $\psi(x) = \{\mathsf{p} \in \mathfrak{P} \mid I(x) \in I(\mathsf{p})\}$. Since, when ψ is not injective, we can replace U

by U/ψ, for simplicity, if we assume ψ is an injection, thus any individual can be identified with its corresponding element[6] in 2 . Then we can formulate a model for logic of relative modality (cf.[1]) as $\mathfrak{M}=<U, \{B^{\mathsf{p}}\}_{\mathsf{p}\in}, V>$, where $B^{\mathsf{p}}=I(\mathsf{p})$ and V is a valuation defined by $V(\mathsf{p}, a)=1$ iff $a\in I(\mathsf{p})$ for a predicate symbol p and a in U. ¿From B^{p}, we can recover a binary relation R^{p} on U by $aR^{\mathsf{p}}b$ iff $b \in B^{\mathsf{p}}$, and thus $\mathfrak{M}$ is a Kripke model. Note that $\|\mathsf{p}\| = I(\mathsf{p})$. For an atomic sentence $\mathsf{p}(\mathsf{c})$ in a monadic predicate logic, we have obviously

$$\mathrm{M} \models \mathsf{p}(\mathsf{c}) \text{ iff } \mathfrak{M}, a \models \mathsf{p}, \text{ (where } I(\mathsf{c})=a.)$$

For two predicate symbols s and p, we have the following two lemmas.

Lemma 4.1 $\mathfrak{M} \models [\mathsf{s}]\mathsf{p}$ iff $B^{\mathsf{s}} \subseteq \|\mathsf{p}\|$ iff $I(\mathsf{s}) \subseteq I(\mathsf{p})$. ∎

Lemma 4.2 $\mathfrak{M} \models <\mathsf{s}>\mathsf{p}$ iff $B^{\mathsf{s}} \cap \|\mathsf{p}\| \neq \emptyset$ iff $I(\mathsf{s}) \cap I(\mathsf{p}) \neq \emptyset$. ∎

For simplicity, we assume that $I : \mathrm{C} \to U$ is a bijection, thus we identify any constant with its corresponding element $I(\mathsf{c})$ in U.

4.2 Representation of Sentences

Consider representation of the four basic types of sentences in a Kripke model.

$$\text{Universal sentences} \begin{cases} \textbf{A} : \text{All } \mathsf{s} \text{ is } \mathsf{p}. \\ \textbf{E} : \text{No } \mathsf{s} \text{ is } \mathsf{p}. \end{cases} \quad \text{Particular sentences} \begin{cases} \textbf{I} : \text{Some } \mathsf{s} \text{ is } \mathsf{p}. \\ \textbf{O} : \text{Some } \mathsf{s} \text{ is not } \mathsf{p}. \end{cases}$$

4.2.1 Universal Sentences and Lower Zooming In

First we consider translation of a universal sentence of type **A** like *'All* human *is* mortal*'* into the above kind of Kripke models. Since Frege's achievement, it is well known that a universal sentence *'All* s *is* p*'* is translated into a predicate logic as $\forall x(\mathsf{s}(x)\to\mathsf{p}(x))$. Given $\mathrm{M}=<U, I>$, because $\mathrm{M} \models \forall x(\mathsf{s}(x)\to\mathsf{p}(x))$ iff $I(\mathsf{s}) \subseteq I(\mathsf{p})$, we have $\mathrm{M} \models \forall x(\mathsf{s}(x) \to \mathsf{p}(x))$ iff $\mathfrak{M} \models [\mathsf{s}]\mathsf{p}$, and thus

Lemma 4.3 *'All* s *is* p*' is true* iff $\mathfrak{M} \models [\mathsf{s}]\mathsf{p}$. ∎

Consider a *zooming in of* $\mathfrak{M}$ *through* $\{\mathsf{s}, \mathsf{p}\}$: $\mathfrak{M}_{\mathsf{s},\mathsf{p}} \stackrel{\mathrm{df}}{=} \mathfrak{I}_{\mathsf{s},\mathsf{p}}(\mathfrak{M})=<U_{\mathsf{s},\mathsf{p}}, \{B^{\mathsf{q}}_{\mathsf{s},\mathsf{p}}\}_{\mathsf{q}\in}, V_{\mathsf{s},\mathsf{p}}>$. Note that $B^{\mathsf{q}}_{\mathsf{s},\mathsf{p}}=\mathfrak{I}_{\mathsf{s},\mathsf{p}}(B^{\mathsf{q}})$. In general, $U_{\mathsf{s},\mathsf{p}}=\{X_1, X_2, X_3, X_4\}$, where $X_1= I(\mathsf{s})\cap I(\mathsf{p})$, $X_2=I(\mathsf{s})\cap I(\mathsf{p})^{\mathrm{C}}$, $X_3=I(\mathsf{s})^{\mathrm{C}}\cap I(\mathsf{p})$, $X_4=I(\mathsf{s})^{\mathrm{C}}\cap I(\mathsf{p})^{\mathrm{C}}$. If $I(\mathsf{s})\subseteq I(\mathsf{p})$, then $X_2=\emptyset$, thus $U_{\mathsf{s},\mathsf{p}}=\{X_1, X_3, X_4\}$. Then $B^{\mathsf{s}}_{\mathsf{s},\mathsf{p}}=\{X_1\}\subseteq\{X_1, X_3\}=\|\mathsf{p}\|^{\mathsf{s},\mathsf{p}}$. The converse is also shown. Hence we have $\mathfrak{M} \models [\mathsf{s}]\mathsf{p}$ iff $\mathfrak{M}_{\mathsf{s},\mathsf{p}} \models [\mathsf{s}]\mathsf{p}$ and thus,

Lemma 4.4 *'All* s *is* p*' is true* iff $\mathfrak{M}_{\mathsf{s},\mathsf{p}} \models [\mathsf{s}]\mathsf{p}$. ∎

[6] Or we identify x with a multisubset of 2 .

Consider further zooming in. We make a quotient set $U_{\mathsf{s}}=U/R^{\mathsf{s}}=\{I(\mathsf{s}), I(\mathsf{s})^{\mathrm{C}}\}$. When $I(\mathsf{s})\subseteq I(\mathsf{p})$, we have $I(\mathsf{s})\subseteq \underline{R}^{\mathsf{s}}(I(\mathsf{p}))$, and thus we can construct the *lower Zooming in* from $\mathfrak{M}_{\mathsf{s}}$ using relative filtration[3].

Definition 4.5 The *lower zooming in* of $\mathfrak{M}_{\mathsf{s},\mathsf{p}}$ through $\{\mathsf{s}\}$ is a tuple $\underline{\mathfrak{M}_{\mathsf{s}}} \overset{\mathrm{df}}{=} \underline{\mathfrak{I}_{\mathsf{s}}^{\mathsf{s},\mathsf{p}}}(\mathfrak{M}_{\mathsf{s},\mathsf{p}})=<U_{\mathsf{s}}, \{B^{\mathsf{q}}_{\mathsf{s}}\}_{\mathsf{q}\in}\ , \underline{V_{\mathsf{s}}}>$, where $\underline{V_{\mathsf{s}}}(\mathsf{p}, X)=1$ iff $X\subseteq \underline{R}^{\mathsf{s}}(I(\mathsf{p}))$, for $X \in U_{\mathsf{s}}$. ■

Lemma 4.6 *'All* s *is* p*' is true* iff $\underline{\mathfrak{M}_{\mathsf{s}}}, I(\mathsf{s}) \models \mathsf{p}$. ■

Example 4.7 Let us consider a structure $<U, I>$ where 'All human is mortal,' that is, $\forall x(\mathsf{human}(x)\rightarrow\mathsf{mortal}(x))$ is true, which means that, in the structure, $I(\mathsf{human})\subseteq I(\mathsf{mortal})$ holds. For the reason of space, we sometimes abbreviate human and mortal as h and m, respectively. We construct a Kripke model $\mathfrak{M}=<U, \{B^{\mathsf{q}}\}_{\mathsf{q}\in}\ , V>$, where V is given, for instance, by the left-hand side of table in Fig.1. Since $B^{\mathsf{h}}=I(\mathsf{human})\subseteq I(\mathsf{mortal})$, we have $\mathfrak{M} \models [\mathsf{human}]\mathsf{mortal}$. Next we make a zooming in of $\mathfrak{M}$ through $\{\mathsf{h}, \mathsf{m}\}$ as $\mathfrak{M}_{\mathsf{h},\mathsf{m}}=<U_{\mathsf{h},\mathsf{m}}, \{B^{\mathsf{q}}_{\mathsf{h},\mathsf{m}}\}_{\mathsf{q}\in}\ , V_{\mathsf{h},\mathsf{m}}>$, where $U_{\mathsf{h},\mathsf{m}}=\{[a_i]_{\mathsf{h},\mathsf{m}}, [b_i]_{\mathsf{h},\mathsf{m}}, [c_i]_{\mathsf{h},\mathsf{m}}\}$ and $V_{\mathsf{h},\mathsf{m}}$ is given in the left-hand side table in Fig.1. Note that $B^{\mathsf{h}}_{\mathsf{h},\mathsf{m}}=\{[a_i]_{\mathsf{h},\mathsf{m}}\}$ and $\|\mathsf{mortal}\|^{\ \mathsf{h},\mathsf{m}}=\{[a_i]_{\mathsf{h},\mathsf{m}}, [b_i]_{\mathsf{h},\mathsf{m}}\}$.

$\mathfrak{M}$		human	mortal $\cdots$
U	a_i	1	1
		$\cdots$	
	a_j	1	1
	b_i	0	1
		$\cdots$	
	b_j	0	1
	c_i	0	0
		$\cdots$	
	c_j	0	0

$\mathfrak{I}_{\{\mathsf{h},\mathsf{m}\}}$ $-\!\!\rightarrow$ Zooming in

$\mathfrak{M}_{\mathsf{h},\mathsf{m}}$		human	mortal $\cdots$
$U_{\mathsf{h},\mathsf{m}}$	$[a_i]_{\mathsf{h},\mathsf{m}}$	1	1
	$[b_i]_{\mathsf{h},\mathsf{m}}$	0	1
	$[c_i]_{\mathsf{h},\mathsf{m}}$	0	0

Fig. 1. Zooming in of $\mathfrak{M}$ through $\{\mathsf{human}, \mathsf{mortal}\}$

Then, we have $\mathfrak{M}_{\mathsf{h},\mathsf{m}} \models [\mathsf{human}]\mathsf{mortal}$. Finally we make the lower zooming in of $\mathfrak{M}_{\mathsf{h},\mathsf{m}}$ through $\{\mathsf{h}\}$ as $\underline{\mathfrak{M}_{\mathsf{h}}}=<U_{\mathsf{h}}, \{B^{\mathsf{q}}_{\mathsf{h}}\}_{\mathsf{q}\in}\ , \underline{V_{\mathsf{h}}}>$, where $U_{\mathsf{h}}=\{I(\mathsf{human}), I(\mathsf{human})^{\mathrm{C}}\}$ with $I(\mathsf{human})= [a_i]_{\{\mathsf{h},\mathsf{m}\}}$ and $I(\mathsf{human})^{\mathrm{C}}=[b_i]_{\mathsf{h},\mathsf{m}}\cup[c_i]_{\mathsf{h},\mathsf{m}}$ and $\underline{V_{\mathsf{h}}}$ is given by the right-hand side table in Fig.2. Since $\underline{R}^{\mathsf{h}}(I(\mathsf{mortal}))=[a_i]_{\mathsf{h},\mathsf{m}}= I(\mathsf{human})$, we have $\underline{\mathfrak{M}_{\mathsf{h}}}, I(\mathsf{human}) \models \mathsf{mortal}$. ■

For a universal sentence of type **E** like *'No* human *is* Flying*'*, we have similar results:

$\mathfrak{M}_{\mathsf{h,m}}$		human	mortal	$\cdots$
$U_{\mathsf{h,m}}$	$[a_i]_{\mathsf{h,m}}$	$\boxed{1}$	$\boxed{1}$	
	$[b_i]_{\mathsf{h,m}}$	0	$\boxed{1}$	
	$[c_i]_{\mathsf{h,m}}$	0	0	

$\underline{\mathfrak{I}^{\mathsf{h,m}}_{\mathsf{h}}}$ $- \rightarrow$ Low er zooming in

$\underline{\mathfrak{M}_{\mathsf{h}}}$		human	mortal	$\cdots$
U_{h}	$I(\mathsf{human})$	$\boxed{1}$	$\boxed{1}$	
	$I(\mathsf{human})^{\mathrm{C}}$	0	0	

Fig. 2. Low er zooming in of$\mathfrak{M}_{\mathsf{h,m}}$ through $\{\mathsf{h}\}$.

Lemma 4.8 *'No* s *is* p*' is true* iff $\mathfrak{M} \models [\mathsf{s}]\neg\mathsf{p}$. ▪

Lemma 4.9 *'No* s *is* p*' is true* iff $\mathfrak{M}_{\mathsf{s,p}} \models [\mathsf{s}]\neg\mathsf{p}$. *(Zooming in)* ▪

Lemma 4.10 *'No* s *is* p*' is true* iff $\underline{\mathfrak{M}_{\mathsf{s}}}, I(\mathsf{s}) \models \neg\mathsf{p}$. *(Lower zooming in)* ▪

4.2.2 Particular Sentences and Upper Zooming In

Next we consider translation of a particular sentence of type **I** lik e *'Some* human *is* genius*'* in to the kind of Kripke models. Also since F rege,it is well known that *'Some* s *is* p*'* is translated into a predicate logic as $\exists x(\mathsf{s}(x) \wedge \mathsf{p}(x))$. Because $\mathfrak{M} \models \exists x(\mathsf{s}(x) \wedge \mathsf{p}(x))$ iff $I(\mathsf{s}) \cap I(\mathsf{p}) \neq \emptyset$, we hav e$\mathfrak{M} \models \exists x(\mathsf{s}(x) \wedge \mathsf{p}(x))$ iff $\mathfrak{M} \models <\mathsf{s}>\mathsf{p}$, and thus

Lemma 4.11 *'Some* s *is* p*' is true* iff $\mathfrak{M} \models <\mathsf{s}>\mathsf{p}$. ▪

F or a*zo oming in of* $\mathfrak{M}$ *through* $\{\mathsf{s},\mathsf{p}\}$, we hav e$\mathfrak{M} \models <\mathsf{s}>\mathsf{p}$ iff $\mathfrak{M}_{\mathsf{s,p}} \models <\mathsf{s}>\mathsf{p}$,

Lemma 4.12 *'Some* s *is* p*' is true* iff $\mathfrak{M}_{\mathsf{s,p}} \models <\mathsf{s}>\mathsf{p}$. ▪

Again let us consider further zooming in. Here we make a quotient set $U_{\mathsf{s}}{=}U/R^{\mathsf{s}} = \{I(\mathsf{s}), I(\mathsf{s})^{\mathrm{C}}\}$, then, b y $I(\mathsf{s}) \cap I(\mathsf{p}) \neq \emptyset$, we hav e $I(\mathsf{s}) \subseteq \overline{R^{\mathsf{s}}}(I(\mathsf{p}))$, and then, we construct the *upper Zooming in* of $\mathfrak{M}_{\mathsf{s,p}}$ through $\{\mathsf{s}\}$.

Definition 4.13 The *upper zooming in* of $\mathfrak{M}_{\mathsf{s,p}}$ through $\{\mathsf{s}\}$ is a tuple $\overline{\mathfrak{M}_{\mathsf{s}}} \stackrel{\mathrm{df}}{=} \overline{\mathfrak{I}^{\mathsf{s,p}}_{\mathsf{s}}}(\mathfrak{M}_{\mathsf{s,p}}) = <U_{\mathsf{s}}, \{B^{\mathsf{q}}_{\mathsf{s}}\}_{\mathsf{q}\in}, \overline{V_{\mathsf{s}}}>$, where $\overline{V_{\mathsf{s}}}(\mathsf{p}, X){=}1$ iff $X \subseteq \overline{R^{\mathsf{s}}}(I(\mathsf{p}))$, for $X \in U_{\mathsf{s}}$.▪

Lemma 4.14 *'Some* s *is* p*' is true* iff $\overline{\mathfrak{M}_{\mathsf{s}}}, I(\mathsf{s}) \models \mathsf{p}$. ▪

Example 4.15 Consider a structure $<U, I>$, where 'Some human is genius,' i.e., $\exists x(\mathsf{human}(x) \wedge \mathsf{genius}(x))$ is true, which means, in the structure, $I(\mathsf{human}) \cap I(\mathsf{genius}) \neq \emptyset$ holds. Then we construct a Kripke model $\mathfrak{M}{=}<U, \{B^{\mathsf{q}}\}_{\mathsf{q}\in}, V>$, where V is given, for instance, by the left-hand side table in Fig.3. Since $B^{\mathsf{h}} \cap \|\mathsf{genius}\| \neq \emptyset$, we hav e$\mathfrak{M} \models <\mathsf{human}>\mathsf{genius}$. For a zooming in of $\mathfrak{M}$ through $\{\mathsf{h},\mathsf{g}\}$ as $\mathfrak{M}_{\mathsf{h,g}}{=}<U_{\mathsf{h,g}}, \{B^{\mathsf{q}}_{\mathsf{h,g}}\}_{\mathsf{q}\in}, V_{\mathsf{h,g}}>$, where $U_{\mathsf{h,g}}{=}\{[a_i]_{\mathsf{h,g}}, [b_i]_{\mathsf{h,g}}, [c_i]_{\mathsf{h,g}}\}$, and $V_{\mathsf{h,g}}$ is given by the right-hand side table in Fig.3. Because $B^{\mathsf{h}}_{\mathsf{h,g}}{=}\{[a_i]_{\mathsf{h,g}}, [b_i]_{\mathsf{h,g}}\}$ and $\|\mathsf{genius})\|^{\mathsf{h,g}}{=}\{[a_i]_{\mathsf{h,g}}, [c_i]_{\mathsf{h,g}}\}$, we hav e $\mathfrak{M}_{\mathsf{h,g}} \models <\mathsf{human}>\mathsf{genius}$. Finally for the upper zooming in of $\mathfrak{M}_{\mathsf{h,g}}$ through $\{\mathsf{h}\}$, i.e., $\overline{\mathfrak{M}_{\mathsf{h}}}{=}<U_{\mathsf{h}}, \{B^{\mathsf{q}}_{\mathsf{h}}\}_{\mathsf{q}\in}, \overline{V}_{\mathsf{h}}>$,

$\mathfrak{M}$		human	genius	$\cdots$
U	a_i	1	1	
			$\cdots$	
	a_j	1	1	
	b_i	1	0	
			$\cdots$	
	b_j	1	0	
	c_i	0	1	
			$\cdots$	
	c_j	0	1	
	d_i	0	0	
			$\cdots$	
	d_j	0	0	

$\mathfrak{I}_{\{\mathsf{h},\mathsf{g}\}}$ $-\rightarrow$ Zooming in

$\mathfrak{M}_{\mathsf{h},\mathsf{g}}$		human	genius	$\cdots$
$U_{\mathsf{h},\mathsf{g}}$	$[a_i]_{\mathsf{h},\mathsf{g}}$	1	1	
	$[b_i]_{\mathsf{h},\mathsf{g}}$	1	0	
	$[c_i]_{\mathsf{h},\mathsf{g}}$	0	1	
	$[d_i]_{\mathsf{h},\mathsf{g}}$	0	0	

Fig. 3. Zooming in of $\mathfrak{M}$ through $\{\mathsf{human}, \mathsf{genius}\}$.

where $U_{\mathsf{h}}=\{I(\mathsf{human}), I(\mathsf{human})^{\mathrm{C}}\}$ with $I(\mathsf{human}) =[a_i]_{\mathsf{h},\mathsf{g}}\cup[c_i]_{\mathsf{h},\mathsf{g}}$ and $I(\mathsf{human})^{\mathrm{C}}$ $=[c_i]_{\mathsf{h},\mathsf{g}} \cup [d_i]_{\mathsf{h},\mathsf{g}}$ and $\overline{V}_{\mathsf{h}}$ is given by the right-hand side table in Fig.4. Hence

$\mathfrak{M}_{\mathsf{h},\mathsf{g}}$		human	genius	$\cdots$
$U_{\mathsf{h},\mathsf{g}}$	$[a_i]_{\mathsf{h},\mathsf{g}}$	1	$\boxed{1}$	
	$[b_i]_{\mathsf{h},\mathsf{g}}$	1	0	
	$[c_i]_{\mathsf{h},\mathsf{g}}$	0	$\boxed{1}$	
	$[d_i]_{\mathsf{h},\mathsf{g}}$	0	0	

$\underline{\mathfrak{I}^{\mathsf{h},\mathsf{g}}_{\mathsf{h}}}$ $-\rightarrow$ (Upper zooming in)

$\overline{\mathfrak{M}}_{\mathsf{h}}$		human	genius	$\cdots$
U_{h}	$I(\mathsf{human})$	1	1	
	$I(\mathsf{human})^{\mathrm{C}}$	0	1	

Fig. 4. Upper zooming in of $\mathfrak{M}_{\{\mathsf{h},\mathsf{g}\}}$ through $\{\mathsf{human}\}$.

we have $\overline{\mathfrak{M}}_{\mathsf{h}}, I(\mathsf{human}) \models \mathsf{genius}$. ∎

For a universal sentence of type **O** like *'Some* human *is not* genius,*'* we have similar results:

Lemma 4.16 *'Some* s *is not* p*' is true* iff $\mathfrak{M} \models \langle \mathsf{s} \rangle \neg \mathsf{p}$. ∎

Lemma 4.17 *'Some* s *is not* p*' is true* iff $\mathfrak{M}_{\mathsf{s},\mathsf{p}} \models \langle \mathsf{s} \rangle \neg \mathsf{p}$. *(Zooming in)* ∎

Lemma 4.18 *'Some* s *is not* p*' is true* iff $\overline{\mathfrak{M}}_{\mathsf{s}}, I(\mathsf{s}) \models \neg \mathsf{p}$. *(Upper zooming in)* ∎

4.3 Conversion

Representation of several conv ersion rules is trivial:

Some s is p.	iff	Some p is s.
$\mathfrak{M}_{\mathsf{s,p}} \models \langle\mathsf{s}\rangle\mathsf{p}$	iff	$\mathfrak{M}_{\mathsf{s,p}} \models \langle\mathsf{p}\rangle\mathsf{s}$
$\overline{\mathfrak{M}}_{\mathsf{s}}, I(\mathsf{s}) \models \mathsf{p}$	iff	$\overline{\mathfrak{M}}_{\mathsf{p}}, I(p) \models \mathsf{s}$

No s is p.	iff	No p is s.
$\mathfrak{M}_{\mathsf{s,p}} \models [\mathsf{s}]\neg\mathsf{p}$	iff	$\mathfrak{M}_{\mathsf{s,p}} \models [\mathsf{p}]\neg\mathsf{s}$
$\underline{\mathfrak{M}}_{\mathsf{s}}, I(\mathsf{s}) \models \neg\mathsf{p}$	iff	$\underline{\mathfrak{M}}_{\mathsf{p}}, I(p) \models \neg\mathsf{s}$

Not (All s is p).	iff	Some s is not p.
$\mathfrak{M}_{\mathsf{s,p}} \models \neg[\mathsf{s}]\mathsf{p}$	iff	$\mathfrak{M}_{\mathsf{s,p}} \models \langle\mathsf{s}\rangle\neg\mathsf{p}$
$\underline{\mathfrak{M}}_{\mathsf{s}}, I(\mathsf{s}) \not\models \mathsf{p}$	iff	$\overline{\mathfrak{M}}_{\mathsf{s}}, I(s) \models \neg\mathsf{p}$

Not (Some s is p).	iff	No s is p.
$\mathfrak{M}_{\mathsf{s,p}} \models \langle\mathsf{s}\rangle\mathsf{p}$	iff	$\mathfrak{M}_{\mathsf{s,p}} \models [\mathsf{s}]\neg\mathsf{p}$
$\overline{\mathfrak{M}}_{\mathsf{s}}, I(\mathsf{s}) \not\models \mathsf{p}$	iff	$\underline{\mathfrak{M}}_{\mathsf{s}}, I(s) \models \neg\mathsf{p}$

4.4 Categorical Syllogism

In contrast with Subsection 4.2, here we take a top-down approach, i.e., without describing the details of an underlying model $\mathfrak{M}$, we simply assume the existence of such basic model so that we can perform reasoning process. F or example, when we are giv ena universal sentence 'All s is p,' we at once construct a model $\mathfrak{M}_{\mathsf{s,p}}$ (or their lo wer model) and we assume it is a result of zooming in of $\mathfrak{M}$ through $\{\mathsf{s},\mathsf{p}\}$ for some $\mathfrak{M}$. There are four basic patterns of syllogism in Aristotle's syllogism such as BARBARA, CELARENT, DARII, and FARIO. Here we illustrate the inference process of the first pattern in our setting. The form of BARBARA and its translation are given b y

BARBARA	(Zooming in)	(Lower zooming in)	
All m is p.	$\mathfrak{M}_{\mathsf{m,p}} \models [\mathsf{m}]\mathsf{p}$	$\underline{\mathfrak{M}_{\mathsf{m}}}, I(\mathsf{m}) \models \mathsf{p}$	$I(\mathsf{m}) \subseteq \underline{R^{\mathsf{m}}}(I(\mathsf{p}))$
All s is m.	$\mathfrak{M}_{\mathsf{s,m}} \models [\mathsf{s}]\mathsf{m}$	$\underline{\mathfrak{M}_{\mathsf{s}}}, I(\mathsf{s}) \models \mathsf{m}$	$I(\mathsf{s}) \subseteq \underline{R^{\mathsf{s}}}(I(\mathsf{m}))$
All s is p	$\mathfrak{M}_{\mathsf{s,p}} \models [\mathsf{s}]\mathsf{p}$	$\underline{\mathfrak{M}_{\mathsf{s}}}, I(\mathsf{s}) \models \mathsf{p}$	$I(\mathsf{s}) \subseteq \underline{R^{\mathsf{s}}}(I(\mathsf{p}))$

First we describe the (simple) zooming in case. By the premises we can assume the follo wing two models:

$\mathfrak{M}^1_{\mathsf{m,p}}$		m p $\cdots$
$U_{\mathsf{m,p}}$	$I(\mathsf{m}) \cap I(\mathsf{p})$	1 1
	$I(\mathsf{m}) \cap I(\mathsf{p})^{\mathrm{C}}$	(*discarded*)
	$I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})$	0 1
	$I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})^{\mathrm{C}}$	0 0

$\mathfrak{M}^2_{\mathsf{s,m}}$		s m $\cdots$
$U_{\mathsf{s,m}}$	$I(\mathsf{s}) \cap I(\mathsf{m})$	1 1
	$I(\mathsf{s}) \cap I(\mathsf{m})^{\mathrm{C}}$	(*discarded*)
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})$	0 1
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})^{\mathrm{C}}$	0 0

where the second rows in each valuation are discarded because $I(\mathsf{m}) \cap I(\mathsf{p})^{\mathrm{C}} = \emptyset$ and $I(\mathsf{s}) \cap I(\mathsf{m})^{\mathrm{C}} = \emptyset$ (we can assume they do not exist b y the premises). T o merge the two models, we make a zooming out of each model through $\{\mathsf{s},\mathsf{m},\mathsf{p}\}$:

		$\mathfrak{M}^1_{\mathsf{s,m,p}} \overset{\mathrm{df}}{=} \mathfrak{O}^{\{\mathsf{m,p}\}}_{\{\mathsf{s,m,p}\}}(\mathfrak{M}^1_{\mathsf{m,p}})$	$\mathfrak{M}^2_{\mathsf{s,m,p}} \overset{\mathrm{df}}{=} \mathfrak{O}^{\{\mathsf{s,m}\}}_{\{\mathsf{s,m,p}\}}(\mathfrak{M}^2_{\mathsf{s,m}})$
		s m p ···	s m p ···
$U_{\mathsf{s,m,p}}$	$I(\mathsf{s}) \cap I(\mathsf{m}) \cap I(\mathsf{p})$	1 1 1	1 1 1
	$I(\mathsf{s}) \cap I(\mathsf{m}) \cap I(\mathsf{p})^{\mathrm{C}}$	(*discarded*)	1 1 0
	$I(\mathsf{s}) \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})$	1 1 0	(*discarded*)
	$I(\mathsf{s}) \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})^{\mathrm{C}}$	1 0 0	(*discarded*)
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m}) \cap I(\mathsf{p})$	0 1 1	0 1 1
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m}) \cap I(\mathsf{p})^{\mathrm{C}}$	(*discarded*)	0 0 1
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})$	0 0 1	0 0 1
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})^{\mathrm{C}}$	0 0 0	0 0 0

By merging them, we have

$\mathfrak{M}^1_{\mathsf{s,m,p}} \circ \mathfrak{M}^2_{\mathsf{s,m,p}}$		s m p
$U_{\mathsf{s,m,p}}$	$I(\mathsf{s}) \cap I(\mathsf{m}) \cap I(\mathsf{p})$	1 1 1
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m}) \cap I(\mathsf{p})$	0 1 1
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})$	0 0 1
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})^{\mathrm{C}}$	0 0 0

to which we again apply zooming in from $\{\mathsf{s},\mathsf{m},\mathsf{p}\}$ to $\{\mathsf{s},\mathsf{p}\}$:

$\mathfrak{M}^1_{\mathsf{s,m,p}} \circ \mathfrak{M}^2_{\mathsf{s,m,p}}$		s	m	p
$U_{\mathsf{s,m,p}}$	$I(\mathsf{s}) \cap I(\mathsf{m}) \cap I(\mathsf{p})$	$\boxed{1}$	1	$\boxed{1}$
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m}) \cap I(\mathsf{p})$	0	1	$\boxed{1}$
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})$	0	0	$\boxed{1}$
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{m})^{\mathrm{C}} \cap I(\mathsf{p})^{\mathrm{C}}$	0	0	0

$\mathfrak{I}^{\{\mathsf{s,m,p}\}}_{\{\mathsf{s,p}\}}$ $- \to$ Zooming in

$\mathfrak{M}^3_{\mathsf{s,p}}$		s	p
$U_{\mathsf{s,p}}$	$I(\mathsf{s}) \cap I(\mathsf{p})$	$\boxed{1}$	$\boxed{1}$
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{p})$	0	$\boxed{1}$
	$I(\mathsf{s})^{\mathrm{C}} \cap I(\mathsf{p})^{\mathrm{C}}$	0	0

where $\mathfrak{M}^3_{\mathsf{s,p}} = \mathfrak{I}^{\{\mathsf{s,m,p}\}}_{\{\mathsf{s,p}\}}(\mathfrak{M}^1_{\mathsf{s,m,p}} \circ \mathfrak{M}^2_{\mathsf{s,m,p}})$. Thus we have

$$\mathfrak{M}^3_{\mathsf{s,p}} \models [\mathsf{s}]\mathsf{p} \quad \text{(and thus } \underline{\mathfrak{M}^3_{\mathsf{s}}}, I(\mathsf{s}) \models \mathsf{p}.)$$

Hence, the process of BARBARA is performed on the basis of the following combination of zooming in & out:

$$\mathfrak{M}^3_{\mathsf{s,p}} = \mathfrak{I}^{\{\mathsf{s,m,p}\}}_{\{\mathsf{s,p}\}}(\mathfrak{O}^{\{\mathsf{m,p}\}}_{\{\mathsf{s,m,p}\}}(\mathfrak{M}^1_{\mathsf{m,p}}) \circ \mathfrak{O}^{\{\mathsf{s,m}\}}_{\{\mathsf{s,m,p}\}}(\mathfrak{M}^2_{\mathsf{s,m}})).$$

Others can be similarly described.

CELARENT	(Zooming in)	(Lower zooming in)	
No m is p.	$\mathfrak{M} \models [\mathsf{m}]\neg\mathsf{p}$	$\mathfrak{P}, \mathsf{m}(\mathfrak{M}), I(\mathsf{m}) \models \neg\mathsf{p}$	$I(\mathsf{m}) \subseteq \underline{R}^{\mathsf{m}}(I(\mathsf{p})^{\mathrm{C}})$
All s is m.	$\mathfrak{M} \models [\mathsf{s}]\mathsf{m}$	$\mathfrak{P}, \mathsf{s}(\mathfrak{M}), I(\mathsf{s}) \models \mathsf{m}$	$I(\mathsf{s}) \subseteq \underline{R}^{\mathsf{s}}(I(\mathsf{m}))$
No s is p	$\mathfrak{M} \models [\mathsf{s}]\neg\mathsf{p}$	$\mathfrak{P}, \mathsf{s}(\mathfrak{M}), I(\mathsf{s}) \models \neg\mathsf{p}$	$I(\mathsf{s}) \subseteq \underline{R}^{\mathsf{s}}(I(\mathsf{p})^{\mathrm{C}})$

DARII	(Bottom-up)	(Top-down)	
All m is p.	$\mathfrak{M} \models [\mathsf{m}]\mathsf{p}$	$\mathfrak{P}, \mathsf{m}(\mathfrak{M}), I(\mathsf{m}) \models \mathsf{p}$	$I(\mathsf{m}) \subseteq \underline{R}^{\mathsf{m}}(I(\mathsf{p}))$
Some s is m.	$\mathfrak{M} \models <\mathsf{s}>\mathsf{m}$	$\mathfrak{P}, \mathsf{s}(\mathfrak{M}), I(\mathsf{s}) \models \mathsf{m}$	$I(\mathsf{s}) \subseteq \overline{R}^{\mathsf{s}}(I(\mathsf{m}))$
Some s is p	$\mathfrak{M} \models <\mathsf{s}>\mathsf{p}$	$\mathfrak{P}, \mathsf{s}(\mathfrak{M}), I(\mathsf{s}) \models \mathsf{p}$	$I(\mathsf{s}) \subseteq \overline{R}^{\mathsf{s}}(I(\mathsf{p}))$

FERIO	(Bottom-up)	(Top-down)	
No m is p.	$\mathfrak{M} \models [\mathsf{m}]\neg\mathsf{p}$	$\mathfrak{P}, \mathsf{m}(\mathfrak{M}), I(\mathsf{m}) \models \neg\mathsf{p}$	$I(\mathsf{m}) \subseteq \underline{R}^{\mathsf{m}}(I(\mathsf{p})^{\mathrm{C}})$
Some s is m.	$\mathfrak{M} \models <\mathsf{s}>\mathsf{m}$	$\mathfrak{P}, \mathsf{s}(\mathfrak{M}), I(\mathsf{s}) \models \mathsf{m}$	$I(\mathsf{s}) \subseteq \overline{R}^{\mathsf{s}}(I(\mathsf{m}))$
Some s is not p	$\mathfrak{M} \models <\mathsf{s}>\neg\mathsf{p}$	$\mathfrak{P}, \mathsf{s}(\mathfrak{M}), I(\mathsf{s}) \models \neg\mathsf{p}$	$I(\mathsf{s}) \subseteq \overline{R}^{\mathsf{s}}(I(\mathsf{p})^{\mathrm{C}})$

5 Concluding Remarks

In this paper, we introduced the two operations of 'zooming in and out' as representing one aspect of granular computing in a logical setting and then applied them in to a formulation of Aristotle's syllogism. In the forthcoming paper, we are planning to extend it to predicative reasoning processes.

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